Towards Diagnosing Inconsistency in Nonmonotonic Multi-Context Systems*

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Abstract. In multi-context systems, heterogeneous contexts interact via nonmonotonic bridge rules. We seek to understand and give reasons for inconsistencies in such systems by means of diagnosis. For this purpose, we propose notions of consistency-based and abduction-based diagnosis, where diagnoses are characterized by sets of bridge rules. Interestingly, the notions of consistency-based and abduction-based diagnoses lead to the same sets of bridge rules which are potentially erroneous.

Introduction. Multi-context systems (MCSs) as defined by Brewka and Eiter [1] are a powerful framework for integrating heterogeneous nonmonotonic logics like ontologies, databases or answer set programs. MCSs can represent inter-contextual information flow and express reasoning with respect to contextual information. The formalism allows for decentralized systems which use point-wise information exchange and consist of multiple components like, for instance, business logics, agents, or knowledge bases in general.

An MCS consists of *contexts* and *bridge rules*. Each context is a knowledge base with an underlying (nonmonotonic) logic providing its semantics in terms of belief sets. Contexts interact through nonmonotonic *bridge rules* of the form

$$(c_s : s) \leftarrow (c_1 : p_1), \ldots, (c_j : p_j), \mathbf{not}\ (c_{j+1} : p_{j+1}), \ldots, \mathbf{not}\ (c_m : p_m). \qquad (1)$$

where $c_s, c_1 \ldots, c_m$ are names of contexts and $s, p_1, \ldots, p_m$ are beliefs of the respective contexts. Intuitively, rule (1) is *applicable* wrt. belief sets $S_1, \ldots, S_m$ of the respective contexts, if $p_i \in S_i$, for $1 \leq i \leq j$, and $p_k \notin S_k$, for $j{+}1 \leq k \leq m$. If the rule is applicable, then its head s is added to the knowledge base of c_s.

Example 1. Assume a health care decision support system which contains the following contexts: a patient history database C_1, a blood and X-Ray analysis database C_2, a description logic ontology of diseases C_3, and a disjunctive logic program implementing an expert system C_4 which suggests proper treatments.

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Consider the following knowledge bases for these contexts, focusing on pneumonia (a lung disease treatable by antibiotics):

$$\begin{aligned}
&C_1 = \{\mathit{allergy_strong_antibiotic}\}\\
&C_2 = \{\neg \mathit{blood_marker}, \mathit{xray_pneumonia}\}\\
&C_3 = \{\mathit{atypical_pneumonia} \sqsubseteq \mathit{pneumonia} \sqcap \mathit{marker}\}\\
&C_4 = \{\mathit{give_s} \vee \mathit{give_w} \leftarrow \mathit{need_antibiotic}.\\
&\qquad \mathit{give_s} \leftarrow \mathit{need_strong}.\\
&\qquad \bot \leftarrow \mathit{give_s}, \mathit{not\ allow_strong}.\\
&\qquad \mathit{nothing_required} \leftarrow \mathit{not\ need_antibiotic}, \mathit{not\ need_strong}.\}
\end{aligned}$$

Contexts C_1 and C_2 provide the information that the patient is allergic to strong antibiotics, that a certain blood marker is not present, and that pneumonia was detected in the X-Ray. C_3 classifies atypical pneumonia as a combination of pneumonia and the presence of a blood marker. C_4 suggests a medication which is either a strong antibiotic s, a weak antibiotic w, or no medication at all.

The bridge rules of the MCS are given as:

$$\begin{aligned}
r_1 &= (3 : \mathit{pneumonia}(p)) &&\leftarrow (2 : \mathit{xray_pneumonia}).\\
r_2 &= (3 : \mathit{marker}(p)) &&\leftarrow (2 : \mathit{blood_marker}).\\
r_3 &= (4 : \mathit{need_antibiotic}) &&\leftarrow (3 : \mathit{pneumonia}(p)).\\
r_4 &= (4 : \mathit{need_strong}) &&\leftarrow (3 : \mathit{atypical_pneumonia}(p)).\\
r_5 &= (4 : \mathit{allow_strong}) &&\leftarrow \mathbf{not}\ (1 : \mathit{allergy_strong_antibiotic}).
\end{aligned}$$

Rules r_1 and r_2 provide input for disease classification to the ontology, r_3 and r_4 link disease information with medication requirements, and r_5 relates acceptance of strong antibiotics with an allergy check on the patient database. □

The semantics of MCSs is defined in terms of *equilibria*. An equilibrium consists of a belief state, i.e., a belief set S_i for each context C_i, which is acceptable for C_i under the addition of beliefs from applicable bridge rules.

Example 2. In our example there exists exactly one equilibrium S, and rules r_1 and r_3 are applicable wrt. S (for C_3, a belief state S_3 consists of all class instances):

$$\begin{aligned}
S = (&\{\mathit{allergy_strong_antibiotic}\}, \{\neg \mathit{blood_marker}, \mathit{xray_pneumonia}\},\\
&\{\mathit{pneumonia}(p)\}, \{\mathit{need_antibiotic}, \mathit{give_w}\}).
\end{aligned}$$ □

Inconsistency in an MCS is the lack of an equilibrium. As the interaction and combination of heterogeneous systems can easily lead to unforeseen and intricate effects, inconsistency is a major—according to our knowledge unaddressed—problem in MCSs. In order to provide support for restoring consistency, we seek to understand and give reasons for inconsistency by means of diagnosis.

Example 3. As a running example, we consider a slightly modified version of Example 1, where the blood serum analysis shows presence of the blood marker:

$$C_2 = \{\mathit{blood\_marker}, \mathit{xray\_pneumonia}\}.$$

This MCS is inconsistent since r_2 and r_4 become applicable, which require that strong antibiotic s is applied. This is in conflict with the patient's allergy. □

We assume that every context is consistent without the influence of bridge rules, therefore we characterize reasons for inconsistency in terms of bridge rules.

Definition 1. *Let M be an MCS and R a set of bridge rules. We write $R \models_M \bot$ to say that the semantics of M using R as its set of bridge rules yields inconsistency. We write $R \not\models_M \bot$ to say that M using R as its set of bridge rules has an equilibrium. In the following, BR denotes the original set of bridge rules of M, and $heads(R)$ denotes the set of heads of rules in R transformed to facts.*

Diagnoses and Explanations for Inconsistency. In nonmonotonic reasoning, forcing rules to be applicable (or forcing them to be not applicable) can cause and prevent inconsistency. For our consistency-based *diagnosis*, we therefore consider pairs of sets of bridge rules, s.t. deactivating the rules in the one set and forcing the rules in the other to be active allows to establish consistency (i.e. an equilibrium) in the system.

Definition 2. *A diagnosis $D^{\pm}$ of an MCS M wrt. BR is a pair $D^{\pm} = (R^-, R^+)$, where $R^-, R^+ \subseteq BR$ and $R^- \cap R^+ = \emptyset$ s.t. $BR \setminus R^- \cup heads(R^+) \not\models_M \bot$.*

Noticeably, this definition resembles the notion of anti-explanation [2], as well as the answer set program debugging approach in [3].

Example 4. Notable diagnoses in our running example are the following:

$$(\{r_1\}, \emptyset),\ (\{r_2\}, \emptyset),\ (\{r_4\}, \emptyset), \text{ and } (\emptyset, \{r_5\}).$$

Accordingly, deactivating r_1, r_2, or r_3, resp. forcing r_5 to be active, will result in a consistent MCS. All other diagnoses are pointwise supersets thereof. □

A research issue are preferred diagnoses wrt. an application domain. Minimality criteria (e.g. applied to preference orderings) can be used for this purpose. For domains where the removal of bridge rules is preferred to the forced activation of unjustified rules, we specialize $D^{\pm}$ to obtain diagnoses of the form $(R^-, \emptyset)$ only. We compare such diagnoses using subset-minimality as preference criterion.

Definition 3. *An s-diagnosis*[1] *D^- of an MCS M wrt. BR is any set $R^- \subseteq BR$ s.t. $BR \setminus R^- \not\models_M \bot$. An s-diagnosis is minimal iff it is minimal wrt. $\subseteq$.*

Example 5. Minimal s-diagnoses in our example are $\{r_1\}$, $\{r_2\}$ and $\{r_4\}$. □

Motivated by abduction-based diagnosis, we consider an *explanation* as a pair of sets of rules whose joint (de-)activation reproduces the observed inconsistency.

Definition 4. *An inconsistency explanation $E^{\pm}$ of an MCS M wrt. BR is a tuple $E^{\pm} = (R^-, R^+)$, where $R^-, R^+ \subseteq BR$ and $R^- \cap R^+ = \emptyset$ s.t. $R^- \cup heads\,(R^+) \models_M \bot$.*

Again we specialize the definition to the first component. In this case we additionally require that an explanation has no consistent superset, to avoid reproducing irrelevant inconsistencies. For instance, the program $\{a \leftarrow not\, a.\}$ is inconsistent under the answer set semantics, but its superset $\{a \leftarrow not\, a.\ \ a.\}$ is consistent.

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[1] The prefix s stands for simple.

Definition 5. *An s-inconsistency explanation* E^+ *of an MCS* M *wrt.* BR *is any set* $R^- \subseteq BR$ *s.t.* $R^- \models_M \bot$ *and there exists no* R' *s.t.* $R^- \subset R' \subseteq BR$ *and* $R' \not\models_M \bot$. *An s-inconsistency explanation is minimal iff it is minimal wrt.* $\subseteq$.

Example 6. The only minimal s-inconsistency explanation E^+ in our running example is $\{r_1, r_2, r_4\}$; its rules are thus necessary to derive inconsistency. □

The union of all minimal diagnoses (explanations) is a set of rules relevant for repairing (causing) inconsistency. Interestingly, these unions coincide (cf. also our example with minimal diagnoses $\{r_1\}$, $\{r_2\}$, $\{r_4\}$, resp. explanation $\{r_1, r_2, r_4\}$).

Theorem 1. *For an inconsistent MCS, the unions of all minimal s-diagnoses* D^-_m *and all minimal s-inconsistency explanations* E^+_m *coincide, i.e.,* $\bigcup D^-_m = \bigcup E^+_m$.

Discussion and Future Work. Minimal D^- diagnoses, i.e., giving preference to rule deactivation, correspond to consistent MCSs, and thus may be used for restoring consistency. However, the resulting options for repair may be too limiting for certain application domains. In our example, the option to disregard the allergy and prescribe strong antibiotics is missed. Rule deactivation either ignores the X-Ray, respectively blood test results, or that atypical pneumonia requires strong antibiotics, with the effect that either the disease is ignored at all, or a medication is suggested which might be to weak.

The study of more specific preference relations between diagnoses is a further research issue in our ongoing project on inconsistency management for MCSs. In addition to an investigation of abstract properties, preferences between diagnoses that emerge from application specific preferences on bridge rules or contexts, e.g., trust levels, are of particular interest. Moreover, we currently investigate approximations of diagnoses given incomplete information (when context knowledge is not fully disclosed), e.g., due to information hiding or security concerns.

Our aim is to provide support for resolving inconsistencies based on preferred diagnoses and inconsistency explanations. Note that due to presence of nonmonotonic contexts (witnessed by context C_4 in our example) the problem is more general than the problem of ontology merging (which may serve to build part of an MCS, however). Moreover, a decision for repair may need to take further domain knowledge into account, as illustrated by our example, where it is not obvious how to resolve the dilemma. Providing a declarative policy language for (semi-)automatic support for repair is the suggestive goal. Also, we plan to apply and evaluate the developed techniques in real world applications.

References

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