FAKULTÄT FÜR !NFORMATIK
Faculty of Informatics

Finding Explanations of Inconsistency in Multi-Context Systems

Thomas Eiter Michael Fink Peter Schüller Antonius Weinzierl

Principles of Knowledge Representation and Reasoning

May 10, 2010

KBS

WWTF
Wiener Wissenschafts-, Forschungs- und Technologiefonds

Vienna University of Technology
Institute for Information Systems
Knowledge-Based Systems Group

Supported by the Vienna Science and Technology Fund (WWTF) under grant ICT08-020

Motivation

- Interlinking and Integrating Knowledge
  - Focus on decentralized systems
  - Heterogeneous and nonmonotonic system parts, here called contexts (databases, ontologies, answer set programs,. . .)
  - Fixed (small) amount of contexts
  - Fixed topology
  - Example: companies linking their business logics

$\Rightarrow$ unifying formalism: Multi-Context Systems

Motivation

- Interlinking and Integrating Knowledge
  - Focus on decentralized systems
  - Heterogeneous and nonmonotonic system parts, here called contexts (databases, ontologies, answer set programs,. . .)
  - Fixed (small) amount of contexts
  - Fixed topology
  - Example: companies linking their business logics

  ⇒ unifying formalism: Multi-Context Systems
- Inconsistencies arise easily, even if all contexts are consistent:
  - Unforseen effects of information exchange
  - Complexity of application and data
- We seek to understand and give reasons for inconsistencies.

Multi-Context Systems

- MCSs introduced by [Giunchiglia & Serafini, 1994]:
  - represent inter-contextual information flow
  - express reasoning w.r.t. contextual information
  - allow decentralized, pointwise information exchange
- Framework extended for integrating heterogeneous non-monotonic logics [Brewka & Eiter, 2007].

Syntax and Semantics of MCSs (1)

- What is a multi-context system?
  - a collection $M = (C_1, \ldots, C_n)$ of contexts
- What is a context?
  - $C_i = (L_i, kb_i, br_i)$
  - L_i: a logic
  - kb_i: the context's knowledge base
  - br_i: a set of bridge rules

Syntax and Semantics of MCSs (1)

- What is a multi-context system?
  - a collection $M = (C_1, \ldots, C_n)$ of contexts
- What is a context?
  - $C_i = (L_i, kb_i, br_i)$
  - L_i: a logic
  - kb_i: the context's knowledge base
  - br_i: a set of bridge rules
- What is a logic?
  - $L = (\mathbf{KB}_L, \mathbf{BS}_L, \mathbf{ACC}_L)$
  - $\mathbf{KB}_L$: set of well-formed knowledge bases
  - $\mathbf{BS}_L$: is the set of possible belief sets
  - $\mathbf{ACC}_L : \mathbf{KB}_L \rightarrow 2^{\mathbf{BS}_L}$: acceptability function:
    Which belief sets are accepted by a knowledge base?

Syntax and Semantics of MCSs (2)

$$M = (C_1, \ldots, C_n) \quad C_i = (L_i, kb_i, br_i) \quad L = (\mathbf{KB}_L, \mathbf{BS}_L, \mathbf{ACC}_L)$$

- What is a belief state?
  $S_i \in \mathbf{BS}_{L_i}$ is a belief set at C_i
  $\Rightarrow S = (S_1, \ldots, S_n)$ is a belief state of M

Syntax and Semantics of MCSs (2)

$$M = (C_1, \ldots, C_n) \quad C_i = (L_i, kb_i, br_i) \quad L = (\mathbf{KB}_L, \mathbf{BS}_L, \mathbf{ACC}_L)$$

- What is a belief state?
  $S_i \in \mathbf{BS}_{L_i}$ is a belief set at C_i
  $\Rightarrow S = (S_1, \ldots, S_n)$ is a belief state of M

- What is a bridge rule?

$$(k : s) \leftarrow (c_1 : p_1), \ldots, (c_j : p_j), \\ \mathbf{not}\ (c_{j+1} : p_{j+1}), \ldots, \mathbf{not}\ (c_m : p_m).$$

  Given a bridge rule r, intuitively...
  ... $(c : p)$ looks at presence of belief p at context C_c (belief set S_c)
  ... r is applicable if positive p_i are present and negative p_i are absent
  ... applicable $\Rightarrow s$ is added to knowledge base of context k

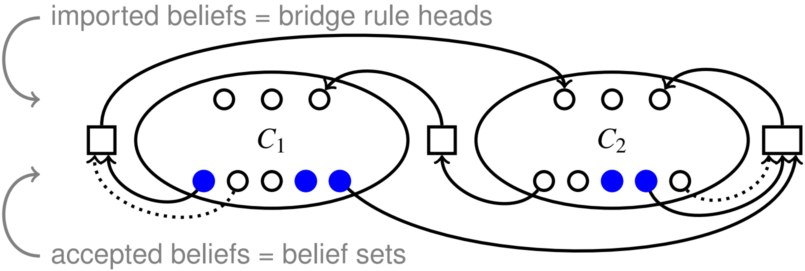
Syntax and Semantics of MCSs (3)

- Equilibrium semantics:

  A belief state $S = (S_1, \ldots, S_n)$

  ... makes certain bridge rules applicable,

  ... so we can add their heads to the kb_i of the contexts.



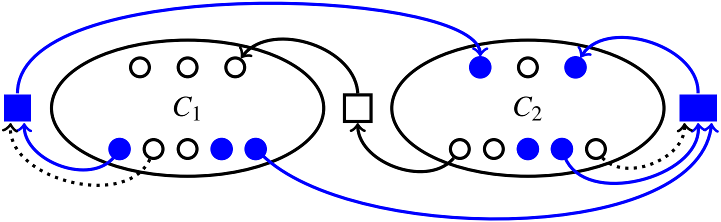
Syntax and Semantics of MCSs (3)

- Equilibrium semantics:

  A belief state $S = (S_1, \ldots, S_n)$

  ... makes certain bridge rules applicable,

  ... so we can add their heads to the kb_i of the contexts.



Syntax and Semantics of MCSs (3)

- Equilibrium semantics:

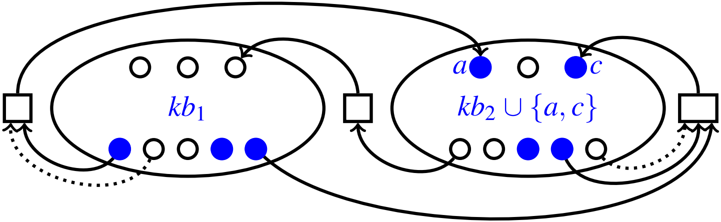
  A belief state $S = (S_1, \ldots, S_n)$

  ...makes certain bridge rules applicable,

  ...so we can add their heads to the kb_i of the contexts.

  S is an equilibrium iff each context plus these heads accepts S_i.

  $\Rightarrow$ Equilibrium condition: $S_i \in \mathbf{ACC}(kb_i \cup H_i)$ for all C_i



Example - Contexts

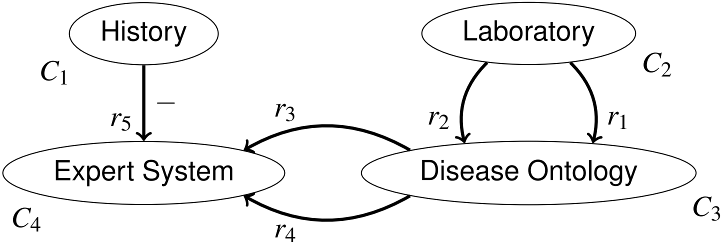
Health care decision support system (wrt. medication and pneumonia):

- patient history database C_1,
- blood and X-Ray analysis database C_2,
- ontology of diseases C_3 (description logic),
- expert system C_4 (disjunctive logic program).

$$
\begin{aligned}
C_1 &= \{allergy\_strong\_ab\}\\
C_2 &= \{\neg blood\_marker, xray\_pneumonia\}\\
C_3 &= \{Pneumonia \sqcap Marker \sqsubseteq AtypPneumonia\}\\
C_4 &= \{give\_strong \lor give\_weak \leftarrow need\_ab.\\
&\quad give\_strong \leftarrow need\_strong.\\
&\quad \bot \leftarrow give\_strong, not\, allow\_strong\_ab.\\
&\quad give\_nothing \leftarrow not\, need\_ab, not\, need\_strong.\}
\end{aligned}
$$

Example - Bridge Rules

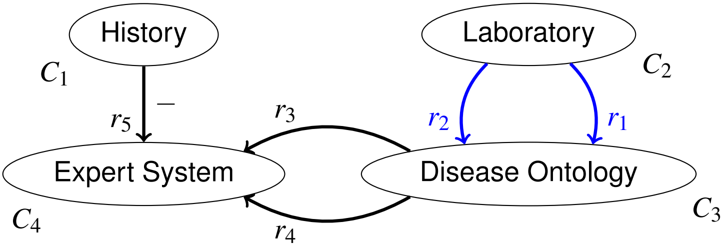
$$
\begin{aligned}
r_1 &= (3 : Pneumonia(p)) &\leftarrow\ & (2 : xray\_pneumonia).\\
r_2 &= (3 : Marker(p)) &\leftarrow\ & (2 : blood\_marker).\\
r_3 &= (4 : need\_ab) &\leftarrow\ & (3 : Pneumonia(p)).\\
r_4 &= (4 : need\_strong) &\leftarrow\ & (3 : AtypPneumonia(p)).\\
r_5 &= (4 : allow\_strong\_ab) &\leftarrow\ & \mathbf{not}\ (1 : allergy\_strong\_ab).
\end{aligned}
$$



$S = (\{allergy\_strong\_ab\}, \{\neg blood\_marker, xray\_pneumonia\},$

Example - Bridge Rules

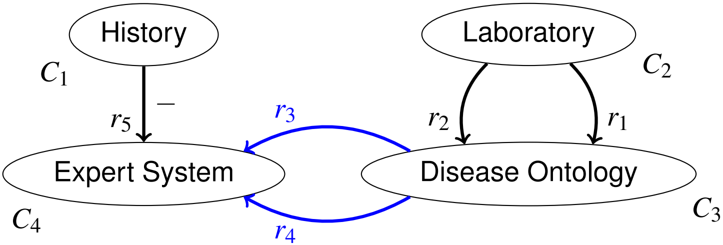
$$
\begin{aligned}
{\color{blue}r_1} &= (3 : \mathit{Pneumonia}(p)) &&\leftarrow (2 : \mathit{xray\_pneumonia}).\\
{\color{blue}r_2} &= (3 : \mathit{Marker}(p)) &&\leftarrow (2 : \mathit{blood\_marker}).\\
r_3 &= (4 : \mathit{need\_ab}) &&\leftarrow (3 : \mathit{Pneumonia}(p)).\\
r_4 &= (4 : \mathit{need\_strong}) &&\leftarrow (3 : \mathit{AtypPneumonia}(p)).\\
r_5 &= (4 : \mathit{allow\_strong\_ab}) &&\leftarrow \mathbf{not}\ (1 : \mathit{allergy\_strong\_ab}).
\end{aligned}
$$



$$
\begin{aligned}
S = (&\{\mathit{allergy\_strong\_ab}\}, \{\neg \mathit{blood\_marker}, \mathit{xray\_pneumonia}\},\\
&\{{\color{blue}\mathit{Pneumonia}(p)}\},
\end{aligned}
$$

Example - Bridge Rules

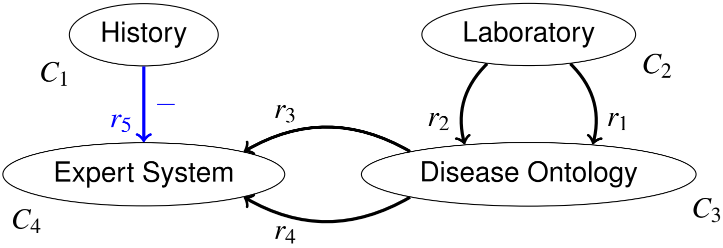
$$
\begin{aligned}
r_1 &= (3 : Pneumonia(p)) &\leftarrow& \ (2 : xray\_pneumonia). \\
r_2 &= (3 : Marker(p)) &\leftarrow& \ (2 : blood\_marker). \\
r_3 &= (4 : need\_ab) &\leftarrow& \ (3 : Pneumonia(p)). \\
r_4 &= (4 : need\_strong) &\leftarrow& \ (3 : AtypPneumonia(p)). \\
r_5 &= (4 : allow\_strong\_ab) &\leftarrow& \ \mathbf{not}\ (1 : allergy\_strong\_ab).
\end{aligned}
$$



$$
\begin{aligned}
S = (&\{allergy\_strong\_ab\}, \{\neg blood\_marker, xray\_pneumonia\}, \\
&\{Pneumonia(p)\}, \{need\_ab,
\end{aligned}
$$

Example - Bridge Rules

$$
\begin{array}{lll}
r_1 = (3 : Pneumonia(p)) & \leftarrow & (2 : xray\_pneumonia). \\
r_2 = (3 : Marker(p)) & \leftarrow & (2 : blood\_marker). \\
r_3 = (4 : need\_ab) & \leftarrow & (3 : Pneumonia(p)). \\
r_4 = (4 : need\_strong) & \leftarrow & (3 : AtypPneumonia(p)). \\
r_5 = (4 : allow\_strong\_ab) & \leftarrow & \mathbf{not}\ (1 : allergy\_strong\_ab).
\end{array}
$$



$S = (\{allergy\_strong\_ab\}, \{\neg blood\_marker, xray\_pneumonia\},$
$\{Pneumonia(p)\}, \{need\_ab, give\_weak\})$ is an equilibrium.

Inconsistency Analysis

- Inconsistency is the lack of an equilibrium.

  We seek to understand and give reasons for inconsistencies.

- We use ideas from model-based diagnosis [Reiter 1987]
- Assumptions:
  - Contexts without input are consistent
  - Bridge rules characterize reasons for inconsistency

Inconsistency Analysis

- Inconsistency is the lack of an equilibrium.

  We seek to understand and give reasons for inconsistencies.

- We use ideas from model-based diagnosis [Reiter 1987]
- Assumptions:
  - Contexts without input are consistent
  - Bridge rules characterize reasons for inconsistency
- Rationale:
  - Context internals are abstracted away – “not our business”
  - Information flow can have unforeseen effects.
  - Knowledge integration between companies:
    changing company knowledge bases (often) impossible

Diagnoses and Explanations

Explaining inconsistency:

- Consistency-based “Diagnosis”:

  Which bridge rules need to be changed to get an equilibrium?

  - “changed” by removing the rule, or
  - “changed” by adding the rule in its unconditional form

  $\Rightarrow$ identifies some rules as "faulty" (causing inconsistency)
  $\Rightarrow$ provides possible repairs

Diagnoses and Explanations

Explaining inconsistency:

- Consistency-based “Diagnosis”:

  Which bridge rules need to be changed to get an equilibrium?

  - “changed” by removing the rule, or
  - “changed” by adding the rule in its unconditional form

  ⇒ identifies some rules as "faulty" (causing inconsistency)
  ⇒ provides possible repairs

- Entailment-based “Inconsistency Explanation”:

  Which bridge rules are required for inconsistency?

  - “required”, assuming all other rules are removed from the MCS

  ⇒ finds groups of rules which *together* cause inconsistency
  ⇒ allows to separate inconsistencies (if there are several of them)

Diagnosis

Diagnosis:

"remove rules, or add them unconditionally, to get consistency"

Definition

A diagnosis is a pair (D_1, D_2), $D_1, D_2 \subseteq br_M$, such that

$$M[br_M \setminus D_1 \cup heads(D_2)] \not\models \bot$$

Notation:

br_M	bridge rules of MCS M
$M[R]$	MCS M with bridge rules R instead of br_M
$M \models \bot$	MCS M is inconsistent
$heads(R)$	rules in R in unconditional form ($\alpha \leftarrow$ for $\alpha \leftarrow \beta$)

Diagnosis

Diagnosis:

"remove rules, or add them unconditionally, to get consistency"

Definition

A diagnosis is a pair (D_1, D_2), $D_1, D_2 \subseteq br_M$, such that

$$M[br_M \setminus D_1 \cup heads(D_2)] \not\models \bot$$

Notation:

br_M	bridge rules of MCS M
$M[R]$	MCS M with bridge rules R instead of br_M
$M \models \bot$	MCS M is inconsistent
$heads(R)$	rules in R in unconditional form ($\alpha \leftarrow$ for $\alpha \leftarrow \beta$)

$D^{\pm}(M)$: set of diagnoses of M

$D^{\pm}_m(M) \subseteq D^{\pm}(M)$: set of pointwise $\subseteq$-minimal diagnoses of M

Example - Diagnoses

Assume $C_2 = \{blood\_marker, xray\_pneumonia\}$

$\Rightarrow$ No equilibrium

Example - Diagnoses

Assume $C_2 = \{blood\_marker, xray\_pneumonia\}$

$\Rightarrow$ No equilibrium

Minimal diagnoses: $(\{r_1\}, \emptyset)$,

- remove r_1 : $(3 : Pneumonia(p)) \leftarrow (2 : xray\_pneumonia)$.

  $\Rightarrow S_3 = \{Marker(p)\}, S_4 = \{give\_nothing\}$

Example - Diagnoses

Assume $C_2 = \{blood\_marker, xray\_pneumonia\}$

$\Rightarrow$ No equilibrium

Minimal diagnoses: $(\{r_1\}, \emptyset)$, $(\{r_2\}, \emptyset)$,

- remove $r_1 : (3 : Pneumonia(p)) \leftarrow (2 : xray\_pneumonia).$

  $\Rightarrow S_3 = \{Marker(p)\}$, $S_4 = \{give\_nothing\}$
- remove $r_2 : (3 : Marker(p)) \leftarrow (2 : blood\_marker).$

  $\Rightarrow S_3 = \{Pneumonia(p)\}$, $S_4 = \{need\_ab, give\_weak\}$

Example - Diagnoses

Assume $C_2 = \{blood\_marker, xray\_pneumonia\}$

$\Rightarrow$ No equilibrium

Minimal diagnoses: $(\{r_1\}, \emptyset)$, $(\{r_2\}, \emptyset)$, $(\{r_4\}, \emptyset)$,

- remove $r_1 : (3 : Pneumonia(p)) \leftarrow (2 : xray\_pneumonia).$

  $\Rightarrow S_3 = \{Marker(p)\}, S_4 = \{give\_nothing\}$
- remove $r_2 : (3 : Marker(p)) \leftarrow (2 : blood\_marker).$

  $\Rightarrow S_3 = \{Pneumonia(p)\}, S_4 = \{need\_ab, give\_weak\}$
- remove $r_4 : (4 : need\_strong) \leftarrow (3 : AtypPneumonia(p)).$

  $$\Rightarrow \begin{array}{l} S_3 = \{Pneumonia(p), Marker(p), AtypPneumonia(p)\} \\ S_4 = \{need\_ab, give\_weak\} \end{array}$$

Example - Diagnoses

Assume $C_2 = \{blood\_marker, xray\_pneumonia\}$

$\Rightarrow$ No equilibrium

Minimal diagnoses: $(\{r_1\}, \emptyset)$, $(\{r_2\}, \emptyset)$, $(\{r_4\}, \emptyset)$, and $(\emptyset, \{r_5\})$.

- remove $r_1 : (3 : Pneumonia(p)) \leftarrow (2 : xray\_pneumonia)$.

  $\Rightarrow S_3 = \{Marker(p)\}$, $S_4 = \{give\_nothing\}$
- remove $r_2 : (3 : Marker(p)) \leftarrow (2 : blood\_marker)$.

  $\Rightarrow S_3 = \{Pneumonia(p)\}$, $S_4 = \{need\_ab, give\_weak\}$
- remove $r_4 : (4 : need\_strong) \leftarrow (3 : AtypPneumonia(p))$.

  $$\Rightarrow \begin{array}{l} S_3 = \{Pneumonia(p), Marker(p), AtypPneumonia(p)\} \\ S_4 = \{need\_ab, give\_weak\} \end{array}$$
- add $r'_5 : (4 : allow\_strong\_ab) \leftarrow$ ~~$not\ (1 : allergy\_strong\_ab)$~~.

  $$\Rightarrow \begin{array}{l} S_3 = \{Pneumonia(p), Marker(p), AtypPneumonia(p)\} \\ S_4 = \{need\_ab, need\_strong, allow\_strong\_ab, give\_strong\} \end{array}$$

Inconsistency Explanation

Inconsistency Explanation:

"rules (heads) that must be present (absent) for inconsistency"

Definition

An inconsistency explanation is a pair (E_1, E_2), $E_1, E_2 \subseteq br_M$, such that

for each pair (R_1, R_2), $E_1 \subseteq R_1 \subseteq br_M$, $R_2 \subseteq br_M \setminus E_2$

$$M[R_1 \cup heads(R_2)] \models \bot$$

$E^{\pm}(M)$ ($E^{\pm}_m(M)$): sets of ($\subseteq$-minimal) inconsistency explanations in M

Inconsistency Explanation

Inconsistency Explanation:

"rules (heads) that must be present (absent) for inconsistency"

Definition

An inconsistency explanation is a pair (E_1, E_2), $E_1, E_2 \subseteq br_M$, such that for each pair (R_1, R_2), $E_1 \subseteq R_1 \subseteq br_M$, $R_2 \subseteq br_M \setminus E_2$

$$M[R_1 \cup heads(R_2)] \models \bot$$

$E^{\pm}(M)$ ($E^{\pm}_m(M)$): sets of ($\subseteq$-minimal) inconsistency explanations in M

Intuition:

- rules in E_1 create inconsistency
- all supersets inconsistent $\Rightarrow$ inconsistency is relevant in M
- adding rules from E_2 unconditionally is necessary to restore consistency

$\Rightarrow$ related to minimal inconsistent sets

Example - Inconsistency Explanations

Assume $C_2 = \{blood\_marker, xray\_pneumonia\}$ (as before)

- Minimal inconsistency explanation: $(\{r_1, r_2, r_4\}, \{r_5\})$.

Example - Inconsistency Explanations

Assume $C_2 = \{blood\_marker, xray\_pneumonia\}$ (as before)

- Minimal inconsistency explanation: $(\{r_1, r_2, r_4\}, \{r_5\})$.
- Minimal diagnoses: $(\{r_1\}, \emptyset)$, $(\{r_2\}, \emptyset)$, $(\{r_4\}, \emptyset)$, and $(\emptyset, \{r_5\})$.

Duality

Assume $C_2 = \{blood\_marker, xray\_pneumonia\}$ (as before)

- Minimal inconsistency explanation: $(\{r_1, r_2, r_4\}, \{r_5\})$.
- Minimal diagnoses: $(\{r_1\}, \emptyset)$, $(\{r_2\}, \emptyset)$, $(\{r_4\}, \emptyset)$, and $(\emptyset, \{r_5\})$.

Theorem

For an inconsistent MCS, the unions of all minimal diagnoses $D_m^{\pm}$ and all minimal inconsistency explanations $E_m^{\pm}$ coincide:

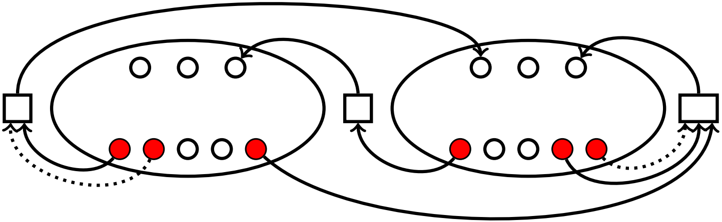
$$\bigcup D_m^{\pm}(M) = \bigcup E_m^{\pm}(M)$$

Notation: $\bigcup X = \left(\bigcup\{A \mid (A,B) \in X\}, \bigcup\{B \mid (A,B) \in X\}\right)$ for X a set of (A,B)

$\Rightarrow$ Diagnoses and explanations identify the same bridge rules

Context Complexity

- recall: equilibrium condition $S_i \in \mathbf{ACC}(kb_i \cup H_i)$ for all C_i
- Output beliefs OUT_i: beliefs in bridge rule body literals

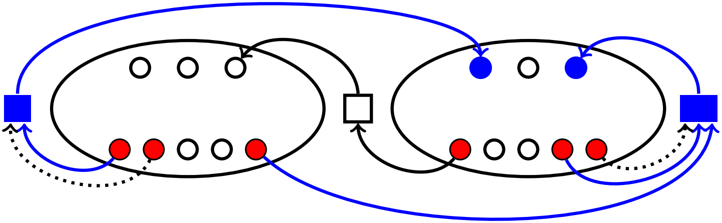


Context Complexity

- recall: equilibrium condition $S_i \in \mathbf{ACC}(kb_i \cup H_i)$ for all C_i
- Output beliefs OUT_i: beliefs in bridge rule body literals
- bridge rules depend on output projected belief sets $S_i' = S_i \cap OUT_i$

⇒ Context complexity = equilibrium existence condition:

$$S_i' \in \mathbf{ACC}_i(kb_i \cup H_i)\big|_{OUT_i}$$



Complexity: Inconsistency Analysis

- Problem: recognition of diagnosis/explanation
- Input: candidate (D_1, D_2) resp. (E_1, E_2) and M

Complexity: Inconsistency Analysis

- Problem: recognition of diagnosis/explanation
- Input: candidate (D_1, D_2) resp. (E_1, E_2) and M

Complexity Results (Completeness):

context complexity	$(D_1, D_2) \overset{?}{\in}$ $D^{\pm}(M)$	$D_m^{\pm}(M)$	$(E_1, E_2) \overset{?}{\in}$ $E^{\pm}(M)$	$E_m^{\pm}(M)$
P	**NP**	$\mathbf{D^P}$	**coNP**	$\mathbf{D^P}$
NP	**NP**	$\mathbf{D^P}$	**coNP**	$\mathbf{D^P}$
$\mathbf{\Sigma_2^P}$	$\mathbf{\Sigma_2^P}$	$\mathbf{D_2^P}$	$\mathbf{\Pi_2^P}$	$\mathbf{D_2^P}$
PSPACE	**PSPACE**			
EXPTIME	**EXPTIME**			

$\mathbf{D^P}$: solve both an **NP** and an independent **coNP** problem

$D^{\pm}$ Computation using HEX-programs

HEX = ASP + Higher order features + external atoms

- Guess diagnosis
- Guess output belief state $\Rightarrow a_i$ atoms
- Evaluate bridge rules $\Rightarrow b_i$ atoms
- Check if output belief state is an output projected equilibrium:

equilibrium condition: $S'_i \in \mathbf{ACC}_i(kb_i \cup H_i)\big|_{OUT_i}$

HEX constraint: $\bot \leftarrow not\ \&con\_out_i[a_i, b_i]().$

- Open source implementation is available:

http://www.kr.tuwien.ac.at/research/systems/dlvhex/mcsiesystem.html

Special Cases and Properties

Special Cases:

- s-Diagnoses:
  "Which rules must be removed to restore consistency?"
- s-Inconsistency Explanations:
  "Which rules must be present to get inconsistency?"

  $\Rightarrow$ duality holds

- "Splitting Sets" on MCS contexts

  $\Rightarrow$ modularity properties

- Preference orders which are different from subset-minimality:

  $\Rightarrow$ duality for certain Ceteris Paribus preference orderings

Explaining Inconsistency — Conclusions

We analyze inconsistencies to know "what's going on".

Our approach...

- uses inconsistency to gain information
- provides possible repairs via diagnoses
- allows to separate sources of inconsistency via explanations

Explaining Inconsistency — Conclusions

We analyze inconsistencies to know "what's going on".

Our approach...

- uses inconsistency to gain information
- provides possible repairs via diagnoses
- allows to separate sources of inconsistency via explanations

We aim at configurable inconsistency management:

- automatic repair may be dangerous (see our example)
- automatic repair may be useful in other cases
- diagnoses and explanations form a basis
  for inconsistency management

Current and Future Work

Current and Future work aims at. . .

- query answers on inconsistent MCSs
  - e.g., defining partial equilibria
  - e.g., defining brave and cautious query answers
- a local point of view to evaluation
  - ⇒ distributed algorithms
- approaches to compare diagnoses/explanations
  - ⇒ quantitative approaches — inconsistency measures
  - ⇒ qualitative approaches — using world knowledge
- implementations and benchmarks
  - relevant application scenarios
  - distributed implementation